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Pullout resistance of single vertical shallow helical and plate anchors in sand

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Abstract This paper presents analytical models to predict the pullout capacity and the load–displacement relationship for shallow single vertical helical and plate anchors in sand. The models were developed based on the failure mechanism deduced from laboratory testing and utilize the limit equilibrium technique. Expression was given to estimate the critical depth for a given anchor/soil conditions, which separates deep from shallow anchors. Furthermore, the radius of influence of a individual anchor on the ground surface is established, and accordingly, the spacing between anchors can be determined to avoid anchors interactions between anchors. The proposed theory compared well with the theories and the experimental data available in the literature.

Résumé Cet article présente un model analytique pour prédire la résistance à l'arrachement et la relation charge-diplacement pour les ancrages à vis et plats verticaux superficiels ancrés dans le sable. Le model est basé sur le mécanisme de rupture déduit des essais de laboratoire et utilise la méthode d'analyse à l'équilibre limite. En outre, une expression a été proposé pour éstimer la profondeur critique pour un ancrage donné permettant d'identifier l'ancrage comme superficiel ou profond. Le rayon d'influence d'un ancrage à la surface du sable autour de l'ancrage a été établi, et par conséquent, l'espacement entre ancrages peut être déterminer pour éviter toute interaction. La théorie proposée montre une bonne concordance avec des résultats theoriques et expérimentaux rapports dans la literature.

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Notation

a	is a constant parameter
a_1	slope of the linear representation of load–displacement of anchors
b	constant
b_1	intercept of the straight lines with ordinate axis
B	diameter of screw anchor
dA	area of an element of the failure surface
E_i	initial tangent modulus
H	embedment depth of anchor

K a parametric constant
K_p' modified coefficient of passive earth pressure
m the exponent determining the rate of variation of E_i with the ratio H/B
Q pullout load on anchor
Q_u ultimate pullout load
r polar coordinate of a given point on the failure surface
R radius of influence failure circle on the sand surface
T resultant shear force acting on the failure surface
T_v vertical component of the resultant shearing resistance acting on the sliding surface
W weight of sand wedge within the failure
Z vertical coordinate of a given point on the failure surface
α angle of inclination of the tangent, of a point on the failure surface, with the horizontal
α_0 the angle made by the curve with the horizontal at $Z = 0$
α_H the angle made by the curve with the horizontal at $Z = H$
Δ displacement of anchor
φ angle of shearing resistance of sand
γ unit weight of the sand
θ angle of rotation of the radius of revolution in a horizontal plane
τ mobilized shear stress along the failure surface

1 Introduction

Foundations subjected to pullout loadings rely heavily on the passive resistance developed on their elements. Anchors are known to be the best foundation elements to provide such resistance. In developing analytical models for these types of foundations depend on identifying a representative failure mechanism of the surrounding soil constitutes one of the major difficulties.

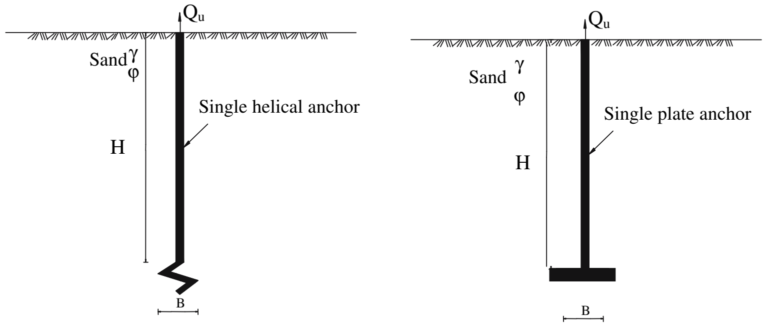
In the literature, empirical relationships were suggested to correlate the pullout capacity of a single anchor to its geometrical properties and the characteristics of the surrounding soil (Udwari et al. 1979; Mitsch and Clemence 1985; Hoyt and Clemence 1989). Furthermore, analytical models were developed to predict the pullout capacity of single anchors in sand based on a assumed failure mechanism; made of a set of planes (Murray and Geddes 1987; Ghaly et al. 1991; Ilamparuthi and Muthukrishnaiah 1999), circular arc surface (Balla 1961; Baker and Kondner 1966; Vesic 1971) and cylindrical surface (Meyerhof and Adams 1968; El Hansy 1980; Mitsch and Clemence 1985). Wang and Wu (1980) developed an upper bound solution based on a failure mechanism consisted of a straight line and a log-spiral. Ghaly and Hanna (1994b) developed theoretical models to predict the pullout capacity of single vertical anchors installed into shallow, transition, and deep depths using the limit equilibrium technique together with Kotter's differential equation to calculate the shear stresses acting on the log-spiral surface. Recently, Dickin and Leung (1990) and Ilamparuthi et al. (2002) have presented a thorough review of the design theories for anchors available in the literature.

Trofimenkov and Mariupolskii (1965) and Healy (1971) proposed a critical depth/diameter ratio of approximately 6 to identify shallow anchors. Furthermore, Vesic (1971) stated that this critical ratio depends on the shear strength of the sand. A rectangular hyperbola has been used to model the load–displacement characteristics of these anchors (Ranjan and Kaushal 1977; El Hansy 1980; Hanna and Ranjan 1992).

Meyerhof and Adams (1968) proposed a value for the base angle (θ_1) of the failure plane with the vertical in the range of $\varphi/4$–$\varphi/2$. While a value of $\varphi/2$ has been proposed by Clemence and Veesaert (1977), Murray and Geddes (1987), and Ilamparuthi et al. (2002). Based on the experimental results, Ghaly et al. (1991) suggested a value for the angle θ_1 as $2\varphi/3$ for shallow anchors. Relatively, fewer suggestions were made for the angle of inclination of the failure surface to the horizontal at the ground level (θ_2). Balla (1961) proposed a value for the angle θ_2 in the range of $\pi/4$ to $\varphi/2$ for a circular failure surface.

2 Theoretical Model

Figure 1 presents single shallow vertical helical and plate anchors in sand subjected to pullout loading. The sand was assumed to be homogeneous, isotropic and behaves as a rigid-perfectly plastic material. In additional, the anchor's helix and plate were assumed to be thin and rigid and in full contact



(a) Helical anchor **(b)** Plate anchor

Fig. 1 Single helical and plate anchors in sand

with the surrounding sand and the frictions between the sand and the tie-rod and the blade surfaces are ignored. The weight of the anchor is considered negligible.

Figure 2 presents the failure mechanism proposed by Ghaly and Hanna (1994a), consisted of logarithmic-spiral rupture surface, Thus:

$$Z = a \cdot \ln(r) + b \tag{1}$$

where Z is the vertical coordinate of a given point at the failure surface, r is the radial coordinate of a given point at the failure surface and a and b are constant parameters. The constant a, depends mainly on the angle of shearing resistance of the sand (φ) and the ratio of depth to diameter of the anchor $[H/B]$. B is the diameter of the helix or plate, H is the embedment depth of the anchor.

Considering the following boundary condition

$$\text{At } Z = 0 \rightarrow r = \frac{B}{2}$$

Thus

$$b = -a \cdot \ln\left[\frac{B}{2}\right] \tag{2}$$

Substituting with the value of b in Eq. 1, thus:

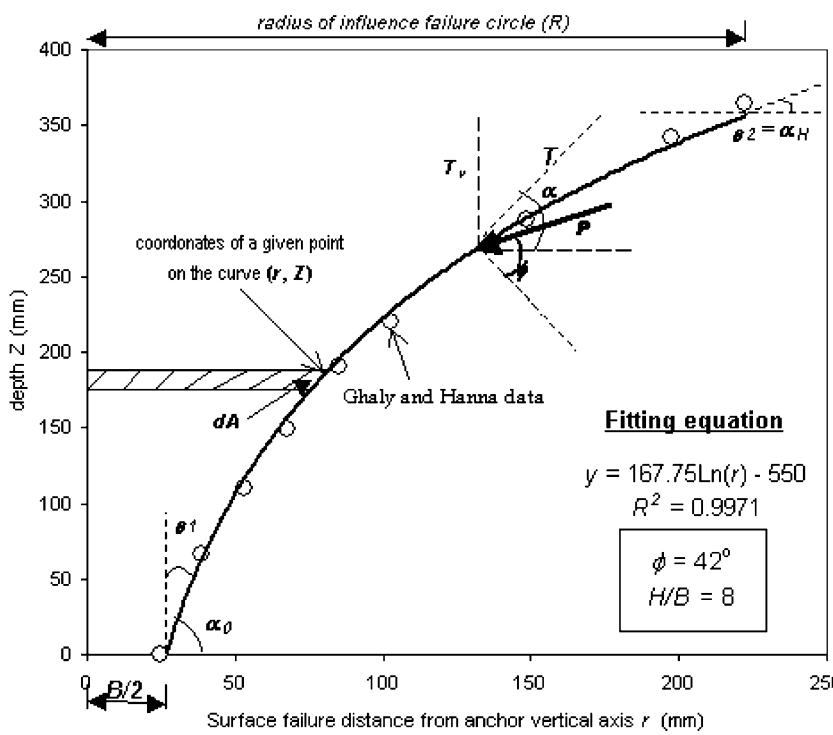


Fig. 2 Typical logarithmic-spiral rupture surface of shallow anchors (Ghaly and Hanna 1994a)

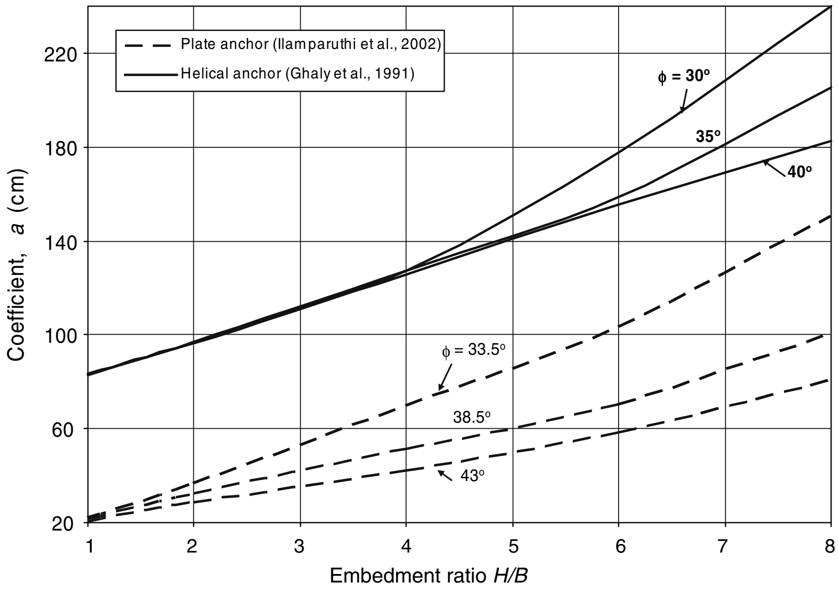


Fig. 3 Coefficient (a) for helical and plate anchors versus the ratio H/B (Eq. 3)

$$Z = a \cdot \ln\left[\frac{2r}{B}\right] \tag{3}$$

The experimental results of Ghaly et al. (1991) were used to calculate the different values of the constant a for helical anchor. These values are given in plotted in Fig. 3. It should be mentioned here that in calculating the coefficient a, the embedded depth of the anchor at failure Z was taken equal to H, ignoring the vertical displacement u of anchor at failure. Preliminary trial calculations showed that the displacement u has no significant effects on the predicted pullout capacity of the anchor.

Referring to Fig. 2, the pullout capacity can be calculated as:

$$Q_u = T_v + W \tag{4}$$

where Q_u is the pullout capacity, T_v is the vertical component of the resultant of the shearing resistance acting on the sliding surface and W is the weight of the sand wedge within the failure surface.

Considering the axisymetrical conditions, thus:

$$W = \int_V \gamma \cdot dV = \int_0^H \left[\int_0^\pi (r^2)\right] \cdot \gamma \cdot d\theta \cdot dz \tag{5}$$

where γ is the unit weight of the sand and θ is the angle of rotation of the radius of revolution in the horizontal plane.

Equation 3 can be rewritten as:

$$r = \frac{B}{2} \cdot e^{\frac{Z}{a}} \tag{6}$$

By substituting Eq. 6 in Eq. 5 and integrating, the following equation can be obtained.

$$W = \frac{\pi}{8} \cdot \gamma \cdot B^2 \cdot a \cdot e^{\frac{2H}{a}} \tag{7}$$

So,

$$W = \bar{A} \cdot e^{\frac{2H}{a}} \tag{8}$$

where

$$\bar{A} = \frac{\pi}{8} \cdot \gamma \cdot B^2 \cdot a$$

Furthermore:

$$T_V = T \cos\left(\frac{\pi}{2} - \alpha\right)$$

Thus

$$T_V = T \sin \alpha$$

where T is the resultant of the shearing forces acting on the failure surface.

For an element dZ, the angle α can be presented as follows, using Eq. 3:

$$\tan\alpha = \frac{dZ}{dr} = \frac{a}{r} \tag{9}$$

where α is the angle of inclination of the tangent, of a point on the failure surface, with the horizontal.

Furthermore,

$$\sin\alpha = \left(\frac{a^2}{a^2+r^2}\right)^{\frac{1}{2}} \tag{10}$$

Applying the MacLaurin series, Eq. 10 can be written as

$$\left(\frac{a^2}{a^2+r^2}\right)^{\frac{1}{2}} = 1 - \frac{r^2}{2a^2} + \varepsilon(r) \tag{11}$$

After several trial calculations, it was noted that the term ε(r) has no significant effect on the value of $\sin\alpha$, accordingly, it was ignored in the present analysis, thus:

$$\sin\alpha = \left(\frac{a^2}{a^2+r^2}\right)^{\frac{1}{2}} \cong 1 - \frac{r^2}{2a^2} \tag{12}$$

Furthermore,

$$\sin\alpha = 1 - \frac{B^2}{8a^2}\cdot e^{\frac{2Z}{a}} \tag{13}$$

The resultant force of the shear stresses, T will be determined by integrating the shear stresses along elementary planes (dA) of the failure surface (Fig. 2) as follows

$$T = \int_A \tau\cdot dA \tag{14}$$

where τ is the mobilized shear stress along the failure surface and dA is the elemental area of a plane element of the failure surface.

Since the analysis was performed on the actual failure surface, the mobilized angle of shearing resistance on the failure surface was taken equal to the angle of shearing resistance of the sand, φ (Hanna 1981), thus:

$$\tau = K'_p\cdot\gamma\cdot(H-Z)\sin\varphi \tag{15}$$

where $K_p{}'$ = is the modified coefficient of passive earth pressure.

The value of $K_p{}'$ varies with depth, depending on the angle of inclination (α). The value of K'_p is equal to the coefficient of passive earth pressure corresponding to a wall inclined at an angle $-\alpha$ (degrees) and having $\frac{\delta}{\varphi} = 1$. From Caquot and Kerisel (1948), the coefficient of passive earth pressure for a wall inclined at $-\alpha$ is expressed as:

$$K'_p = k_1\cdot(\cot\alpha)^{k_2} \tag{16}$$

where k_1, k_2 are constants that depend on the angle of shearing resistance (φ). The values of k_1 and k_2 are calculated and given in Table 1 for different values of φ.

From Eq. 9, for a given depth,

$$\cot\alpha = \frac{r}{a} \tag{17}$$

Therefore,

$$K'_p = k_1\left(\frac{r}{a}\right)^{k2} \tag{18}$$

Furthermore,

$$K'_p = k_1\left(\frac{B}{2a}\right)^{k2}\cdot e^{k2\frac{Z}{a}} \tag{19}$$

Thus:

$$T_V = \int_A K'_p\cdot\gamma\cdot(H-Z)\cdot\sin\alpha\cdot\sin\varphi\cdot dA \tag{20}$$

Furthermore,

Table 1 Values of k_1 and k_2 for different values of angle of shearing (φ)

φ (degree)	k_1	k_2	R^2
10	1.7478	−0.0037	
15	2.8307	−0.0886	0.9826
20	4.8251	−0.183	0.9688
25	8.6965	−0.2844	0.9534
30	16.753	−0.3934	0.9451
35	35.066	−0.5109	0.939
40	81.697	−0.64	0.938
45	220.7	−0.7842	0.9373

$$T_V = \int_0^H \int_0^{2\pi} \left(\frac{1}{2}B \cdot e^{\frac{Z}{a}}\right) \cdot k_1 \left(\frac{B}{2a}\right)^{k2} \cdot e^{k2\frac{Z}{a}} \cdot \gamma \cdot (H - Z) \cdot \left[1 - \frac{B^2}{8a^2} \cdot e^{\frac{2Z}{a}}\right] \cdot \sin\varphi \cdot d\theta \cdot dz. \quad (21)$$

The vertical component T_v can be rewritten as:

$$T_v = K_1 I_1 - K_2 I_2 \quad (22)$$

where:

$$K_1 = \frac{1}{2}B \cdot k_1 \left(\frac{B}{2a}\right)^{k2} \cdot \gamma \cdot \sin\varphi \quad (23)$$

$$K_2 = \frac{1}{16}\frac{B^3}{a^2} \cdot k_1 \left(\frac{B}{2a}\right)^{k2} \cdot \gamma \cdot \sin\varphi \quad (24)$$

$$I_1 = \int_0^H \int_0^{2\pi} e^{(k_2+1)\frac{Z}{a}} \cdot (H - Z) \cdot d\theta \cdot dZ \quad (25)$$

$$I_2 = \int_0^H \int_0^{2\pi} e^{(k_2+3)\frac{Z}{a}} \cdot (H - Z) \cdot d\theta \cdot dZ \quad (26)$$

Integrating Eqs. 25 and 26 gives:

$$I_1 = 2\pi \cdot \frac{a^2}{(k_2+1)^2} \cdot e^{(k_2+1)\frac{H}{a}} - 2\pi\left(\frac{a^2}{(k_2+1)^2} + \frac{a}{(k_2+1)} \cdot H\right) \quad (27)$$

$$I_2 = 2\pi \cdot \frac{a^2}{(k_2+3)^2} \cdot e^{(k_2+3)\frac{H}{a}} - 2\pi\left(\frac{a^2}{(k_2+3)^2} + \frac{a}{(k_2+3)} \cdot H\right) \quad (28)$$

Substituting Eqs. 23, 24, 27 and 28 in Eq. 22 yields:

$$T_V = \bar{B} \cdot e^{(k_2+1)\frac{H}{a}} - \bar{C} \cdot e^{(k_2+3)\frac{H}{a}} + \bar{D} \quad (29)$$

where

$$\bar{B} = \pi \cdot B \cdot k_1 \cdot \left(\frac{B}{2a}\right)^{k_2} \cdot \frac{a^2}{(k_2+1)^2} \cdot \gamma \cdot \sin\varphi \quad (30)$$

$$\bar{C} = \frac{1}{8}\pi \cdot k_1 \cdot \left(\frac{B}{2a}\right)^{k_2} \cdot \frac{B^3}{(k_2+3)^2} \cdot \gamma \cdot \sin\varphi \quad (31)$$

$$\bar{D} = \frac{1}{8}\pi \cdot k_1 \cdot \left(\frac{B}{2a}\right)^{k_2} \cdot \frac{B^3}{a^2} \cdot \gamma \cdot \sin\varphi \cdot \left(\frac{a^2}{(k_2+3)^2} + \frac{a}{(k_2+3)} \cdot H\right) - \pi \cdot B \cdot k_1 \cdot \left(\frac{B}{2a}\right)^{k_2} \cdot \gamma \cdot \sin\varphi \cdot \left(\frac{a^2}{(k_2+1)^2} + \frac{a}{(k_2+1)} \cdot H\right) \quad (32)$$

From Eqs. 8 and 29, Eq. 33 can be driven as:

$$Q_u = \bar{A} \cdot e^{\frac{2H}{a}} + \bar{B} \cdot e^{(k_2+1)\frac{H}{a}} - \bar{C} \cdot e^{(k_2+3)\frac{H}{a}} + \bar{D} \quad (33)$$

Equation (33) predicts the pullout capacity of these anchors.

The experimental data of the pullout load–displacement curves (Q versus Δ) of Ghaly et al. (1991) and Ghaly and Hanna (1994a) were examined following the presentation proposed by Hanna and Ranjan (1992). After some trial, it was found that experimental pullout load–displacement curves can be well represented by a hyperbolic relationship. Furthermore, the ratio of displacement (Δ) to the applied pullout load (Q) versus the anchor's displacement was found to be almost linear (Fig. 4). Thus, the following equation can be developed:

$$\frac{\Delta}{Q} = a_1\Delta + b_1 \quad (34)$$

where Q is the applied pullout load on the anchor (N), Δ is the corresponding anchor displacement (mm), a_1

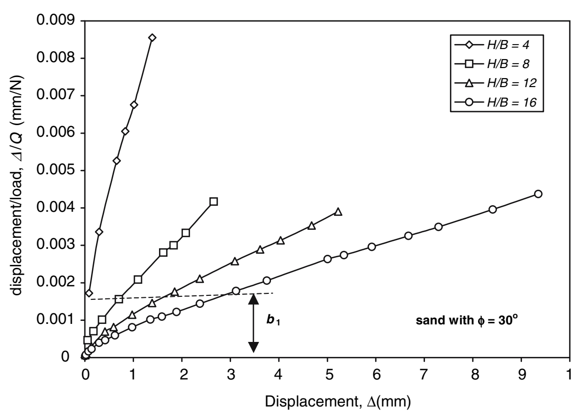


Fig. 4 Typical variation of the ratio of displacement/load versus displacement of shallow anchor in sand (from Ghaly et al. 1991)

is the slope of the load–displacement of the anchor ($a_1 = (1/Q_u)$), b_1 is the intercept of the straight line with the (Y) coordinate axis ($b_1 = (1/E_i)$) (Fig. 4) and E_i is the initial stiffness, which depends on ϕ and the ratio (H/B) (expressed in N/mm).

The transformed hyperbolic form of Eq. 34 can be presented as follows:

$$Q = \frac{\Delta}{a_1\Delta + b_1} \tag{35}$$

Equation 35 can be rewritten as:

$$Q = Q_u \cdot \frac{\Delta}{\Delta + K} \tag{36}$$

where

$$K = \frac{Q_u}{E_i} \tag{37}$$

K is a constant expressed in mm, which depends mainly on the angle of shearing resistance of the sand (φ) and the ratio (H/B).

The experimental data of Ghaly et al. (1991), was used to develop an expression of the parameter E_i in terms of φ and the ratio (H/B). The values of b_1 obtained from these results (Eq. 34), were plotted in Fig. 5 against the ratio (H/B) for different values of φ. From this Figure, the initial stiffness E_i can be expressed as follows:

$$b_1 = \frac{1}{E_i} = 0.018\left(\frac{H}{B}\right)^{-m} \tag{38}$$

where m is the exponent, which determine the rate of variation of E_i with the ratio (H/B) (dimensionless value). The values of m produced by Eq. 38 were plotted against tan (φ) in Fig. 6. Based on this figure, the exponent m can be expressed as:

$$m = 1.3\tan\varphi + 0.93. \tag{39}$$

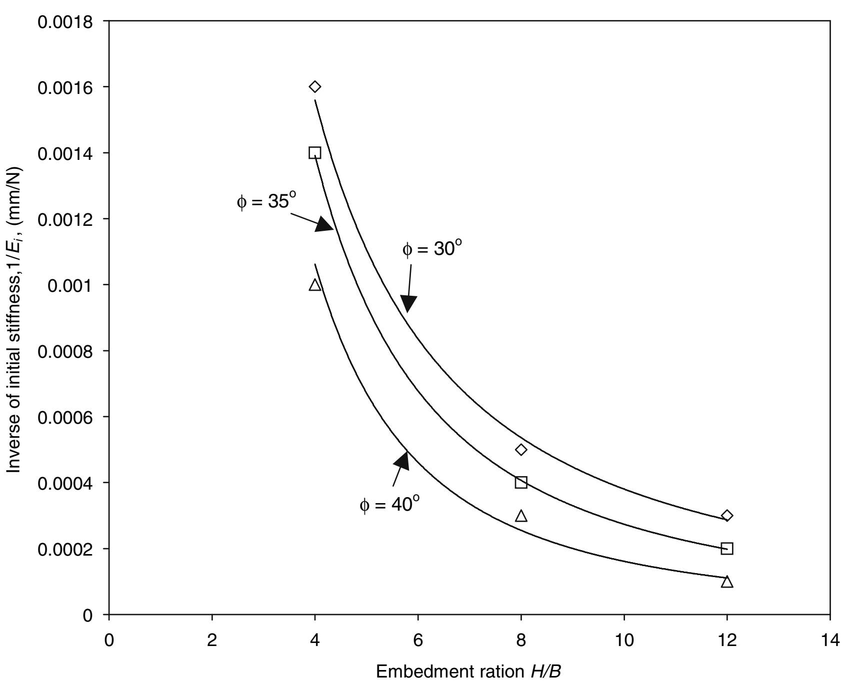
It should be noted that Eq. 38 is similar to the relationship proposed by Duncan et al. (1980) between the tangent modulus and the confining pressure for soil. By substituting Eq. 38 in Eq. 37, the following equation can produced:

$$K = 0.018 \cdot Q_u\left(\frac{H}{B}\right)^{-m} \tag{40}$$

3 Results

Equation 33 was used to calculate the pullout capacity Q_u for different values of (H/B) ratios and

Fig. 5 Variation of the inverse of initial stiffness ($1/Ei$) with the ratio H/B



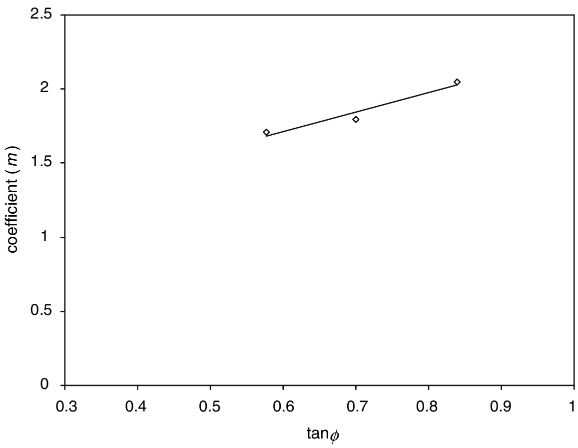


Fig. 6 Variation of the exponent (m) with tan φ

angle of shearing resistance of the sand (φ). The pullout capacity factor N_{qu} was then calculated using the following equation,

$$N_{qu} = \frac{Q_u}{\gamma A H} \tag{41}$$

where Q_u is the ultimate pullout load, γ is the unit weight of the sand, A is the surface area of anchor and H is the installation depth.

Figure 7a and b present the values of the pullout capacity factor N_{qu} versus the embedment ratio (H/B) as function of the angle of shearing resistance φ for helical and plate anchors, respectively. It should be mentioned herein that the values of the constant a (Eq. 3) for plate anchors were deduced by trials and error from Eq. 33 using the pullout capacity results of Ilamparuthi et al. (2002). The different values of this constant for plate anchors were also given in Fig. 3. It can be noted from these figures (Fig. 7a, b) that the pullout capacity increases with an increase of the angle φ and the ratio (H/B) in a hyperbolic form. Figure 7a and b can be regarded as design charts to predict the ultimate pullout load of vertical shallow helical and plate anchors embedded in sand.

Ghaly et al. (1991) have proposed values for the critical depth (H_c), which distinguish between deep and shallow anchors. These values are presented in graphical form in Fig. 8 and presented in this study in Eqs. 42 and 43 for shallow and deep anchors, respectively.

For shallow anchors

$$H < H_c = B \cdot (0.4\varphi - 5) \tag{42}$$

For deep anchors

$$H > B \cdot (0.4\varphi - 3) \tag{43}$$

Between these two limits, the anchor is considered in a transit zone (Ghaly et al. 1991).

Considering that:

$$\text{At} Z = H \rightarrow r = R$$

where, R is the radius of the influence circle in the sand mass at the ground surface (Fig. 2).

The values of R can be determined, from Eq. 3, as follows,

$$R = \frac{B}{2} \cdot e^{\frac{H}{a}} \tag{44}$$

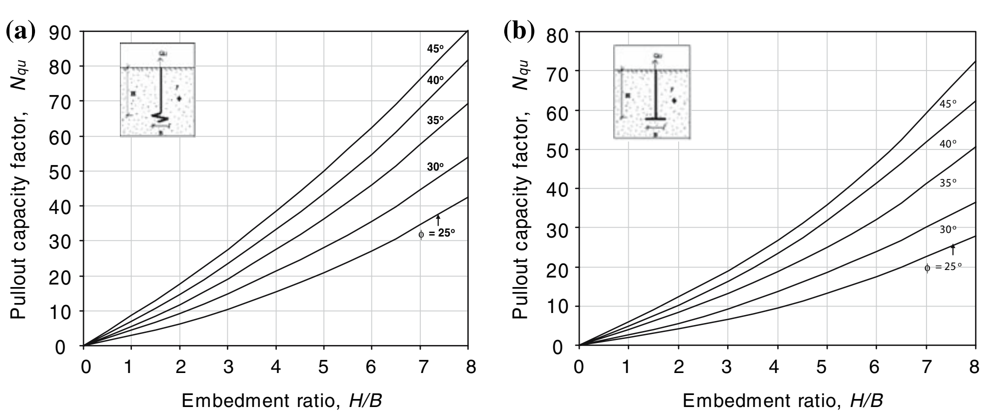


Fig. 7 (**a**) Design chart for pullout capacity factor (N_{qu}) computation (*helical anchors*). (**b**) Design chart for pullout capacity factor (N_{qu}) computation (*plate anchors*)

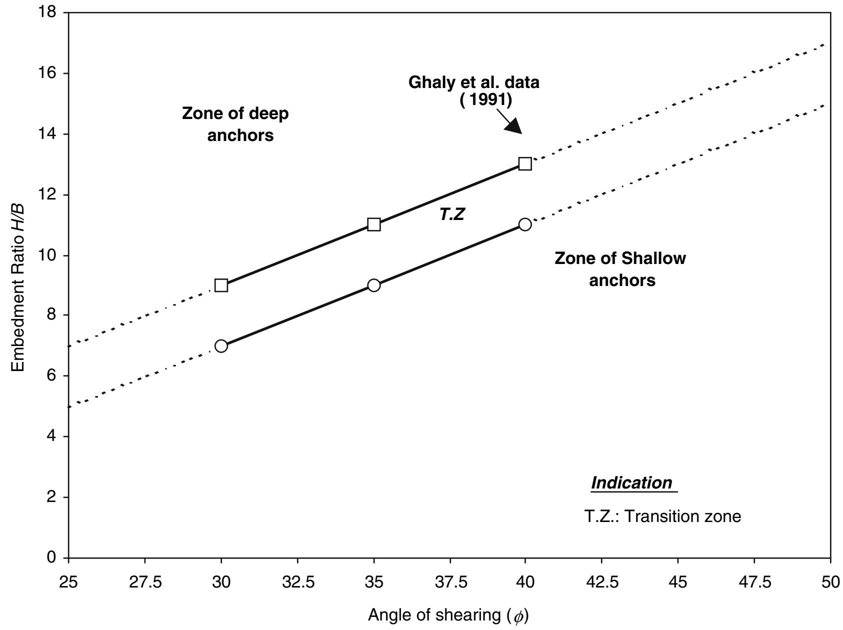


Fig. 8 Variation of critical height (H_c) with the angle of shearing (φ) and the diameter (B)

These values are useful to establish the active zone of each anchor, and according the overlapped areas in the group.

The angles θ_1 and θ_2, which were defined previously (Fig. 2), can be expressed as:

$$\theta_1 = \frac{\pi}{2} - \alpha_0 \tag{45}$$

$$\theta_2 = \alpha_H \tag{46}$$

where α_0 is the angle of the curve with the horizontal at $Z = 0$ and α_H is the angle of the curve with the horizontal at $Z = H$. Therefore:

$$\cos\theta_1 = \sin\alpha_0 = \left(\frac{1}{1+\left(\frac{B}{2a}\right)^2}\right)^{\frac{1}{2}} \cong 1 - \frac{1}{2}\left(\frac{B}{2a}\right)^2 \tag{47}$$

$$\sin\theta_2 = \sin\alpha_H = \left(\frac{1}{1+\left(\frac{B}{2a}\right)^2 \cdot e^{\frac{2H}{a}}}\right)^{\frac{1}{2}} \cong 1 - \frac{1}{2}\left(\frac{B}{2a}\right)^2 \cdot e^{\frac{2H}{a}} \tag{48}$$

4 Comparison Between Theoretical and Experimental Results

In this section, the results predicted by the proposed theory are compared with the laboratory and field test results available in the literature for the cases of vertical shallow plate and single and multi helix anchors in sand as well as other theories.

The pullout capacity for belled piles is often determined from the theories developed for plate anchor (Ilamparuthi et al. 2002). The available field data on belled piles will be also used to validate the theory developed herein. Furthermore, the predicted values by the present theory will be compared with the values produced by other theories available in the literature.

Tables 2 and 3 present the predicted values of the pullout capacity factors, N_{qu}, together with the results of laboratory tests performed on plate and helical anchors installed in loose to medium-dense and dense sands, respectively. It can be noted from Table 2 that the proposed theory predicts the values of N_{qu} within 5–22% of the experimental results for plate anchors and within 2–21% for the helical anchors, having the majority of these results below or around the average of these ranges. While for dense sand (Table 3) the proposed theory generally overestimates the results notably for lower values of H/B. This is due to mainly the dilatation effect generated in the sand mass at shallow depths during anchors installation. In order to overcome these persistent problems, trial calculations were performed using the modified friction angle φ^* proposed by Davis (1968), Drescher and Detournay (1993) and Michalowski (1997) as follows:

Table 2 Comparison between the predicted values of the pullout capacity factor from present theory with available laboratory test results (loose-medium dense sand)

Reference	B(mm)	H/B	$\varphi(^{\circ})$	Measured N_{qu}	Predicted N_{qu}	Deviation (%)
Ilamparuthi et al. (2002) (*plate anchor*)	100–400	2	33.5	7.6	6.7	11.8
		4		17.3	20.2	14.3
		6		29.6	30	1.3
		8		46.4	40.8	12.0
		2	38.5	9.6	9.4	2.1
		4		22.7	22.3	1.7
		6		39.9	37	7.2
		8		60.6	63	3.8
Fadl (1981) (*plate anchor*)	75	4	36.5	20.2	21.5	6.0
		6		34.8	28.6	17.8
		8		54.2	42	22.5
		4	41.5	24.8	27	8.1
		6		44.1	46	4.1
		8		66	70	5.7
Andreadis and Harvey (1981) (*plate anchor*)	N.A.	4	38	26.2	25	4.6
		6		37.6	38	1.1
		8		57.7	51	11.6
Balla (1961) (*plate anchor*)	60–120	2	38	9.5	12	20.8
		4		21.6	23	6.1
		6		37.7	40	5.7
Ghaly et al. (1991) (*Single helix*)	50	2	30	12.7	15	15.3
		4		21.9	27	18.8
		6		35.6	42	15.2
		8		53.9	51	5.4
		2	35	12.6	16	21.2
		4		27.8	35	20.6
		6		46.6	59	21.0
		8		69.3	77	10
		2	40	14.6	18	17.8
		4		33.2	40.5	18.0
		6		54.9	68	19.2
Ghaly and Clemence (1998) (*Single helix*)	50	4	31	22.7	21	7.5
		8		56.9	49.5	13.0
		4	36	28.6	30	4.7
		8		71.8	73	1.6

$$\tan \varphi * = \frac{\cos \psi \sin \varphi}{1 - \sin \psi \sin \varphi} \quad (49)$$

where ψ is the dilatancy angle of sand, which is approximately equal to $\varphi - 30\,\text{deg}$ (Bolton 1986). The predicted values of N_{qu} for dense sand using the modified friction angles are also given in Table 3, where better agreement can be noted, especially for the cases of $H/B < 6$, which represent the practical range for shallow anchors.

The field data available in the literature for the pullout capacity of plate, single and multi helical anchors and belled piles are used to validate further the present theory. The results of this comparison are presented in Table 4 where good agreement can be noted (for dense sand, $\varphi \geq 40\,\text{deg}$, calculations were

Table 3 Comparison between the predicted values of the pullout capacity factor from present theory with available laboratory test results (dense sand)

Reference	B (mm)	H/B	$\varphi(^{\circ})$	$\varphi*(^{\circ})$ Eq. 49	N_{qu} *Measured*	N_{qu} (present theory) *Predicted* with φ	*Predicted* with $\varphi*$	*Deviation* (%)
Baker and Kondner (1966) (*plate anchor*)	76	2	42	38	7	12	9.3	24.7
		4			18	26	21	14.3
		6			42	45	39	7.1
Murray and Geddes (1987) (*plate anchor*)	50.8–88.9	4	44	39.3	22	26	23	4.3
		6			40	46	41	2.4
		8			70	72	62	11.4
Bemben and Kupferman (1975) (*plate anchor*)	76–152	2	46	40.5	12	13	10.5	12.5
		3.5			30	24	23.5	21.6
		5			50	37	34	32
Ilamparuthi et al. (2002) (*plate anchor*)	100–400	3.91	43	38.7	18.98	25	22	13.7
		5.99			45.04	44.5	39	13.4
		6.91			48.36	57	50	3.3
Sutherland et al. (1982) (*Single helix*)	25–76	3.9	41.5	37.7	19	23.5	21	9.5
Ghaly and Clemence (1998) (*Single helix*)	50	4	42	38	38.1	36	33	13.4
		8			98.7	85	79	19.9

performed using the modified friction angle $\varphi*$ defined by Eq. 49).

The comparisons presented in Tables 2, 3 and 4 are also presented in graphical forms in Fig. 9, where good agreement between the measured and the predicted values can be noted. Although these results are within the acceptable range in the field of geotechnical engineering, the discrepancies reported could be due to the disturbance in the sand mass which takes place during installation, sand dilatancy (Rowe and Davis 1982), overconsolidation of the sand (Hanna and Carr 1971) and the flexibility of the anchor's helix or plate (Abdel Rahman et al. 1992).

In additional, the values of the pullout capacity factor predicted by the proposed theory are also compared with those produced by other theories available in the literature. The results of this comparison are presented in Table 5, where good agreement can be found with the theory of Ghaly and Hanna (1994b) for helical anchors and Murray and Geddes (1987) for plate anchors. The discrepancy noted between the results of the present theory and

Table 4 Comparison between the pullout capacity factor from present theory with available field results

Reference	H (m)	H/B	$\varphi(^{\circ})$	Measured N_{qu}	Present theory N_{qu}	Deviation (%)
Sutherland et al. (1982) (*Single helix*)	4.572	1.91	42 ($\varphi* = 37.3$ deg)	17.5	14.7	16.0
	5.18	2.17	42 ($\varphi* = 37.3$ deg)	19.3	15.6	19.1
	7.0	2.94	35	22.8	23.1	1.3
Mitsch and Clemence (1985) (*Multi helix*)	–	4	45 ($\varphi* = 39.9$ deg)	52	36.6	29.4
	–	6	45 ($\varphi* = 39.9$ deg)	73	58.3	20.1
Kananyan (1966) (Plate anchor)	3.48	8.56	37	48	57	15.8
Kwasniewski et al. (1975) (Plate anchor)	1.90	2.71	28	6.93	6.8	1.9
Downs and Chieurzzi (1966) (*Belled pile*)	2.77	3.71	24.7	9.13	8.6	5.8

$\varphi*$ was deduced from Eq. 49

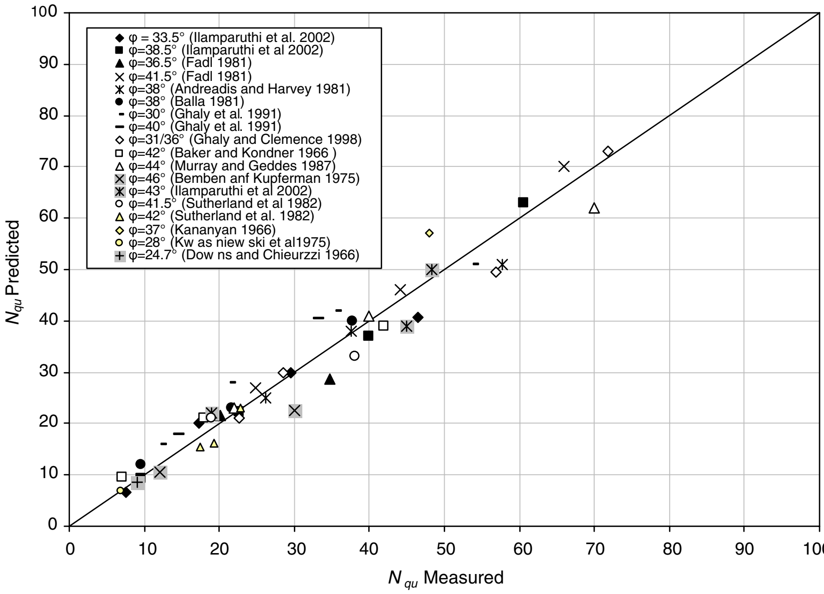


Fig. 9 Comparison between the measured and the predicted values of the pullout capacity factor (laboratory and field data)

other theories can be attributed mainly to the assumptions used in developing these theories. Likewise, the predicted values of N_{qu} for dense sand were computed with the modified friction angle $\varphi*$.

Figure 10 presents comparison between the load–displacement curves produced by the present theory and those available in the literature, where good agreement can be found.

5 Design Procedure

The following procedure is recommended to determine the ultimate pullout load and the load–displacement characteristics of a single vertical shallow helical and plate anchors in sand:

1. Given the angle of shearing resistance of the sand (φ) and the base diameter (B) of the anchor helix

Table 5 Comparison between the pullout capacity factor of the present study and the results produced by other theories

H/B	$\varphi(^\circ)$	Predicted N_{qu}									
		Present theory		Balla (1961)	Vesic (1965)	Mariupol'skill (1965) *H/B* = 2, 3 & 4 (P)	Matsuo (1967)	Meyerhof and Adams (1968) (P)	Saeedy (1987)	Murray and Geddes (1987) (P)	Ghaly and Hanna (1994b) (H)
		(H)	(P)	(P)	(P)		(P)		(P)		
3.0	25	9.9	7.1	6.3	3.4	2.0	5.3	4.6	3.1	7.52	10.1
	35	19.3	13.6	7.1	4.5	3.2	7.1	9.3	6.5	10.97	12.5
	45 ($\varphi* = 39.9$)	24.1	16.4	8.1	5.5	4.7	9.8	13.7	9.8	14.53	20.1
5.0	25	21.3	13.4	7.9	6.8	2.6	9.5	12.4	8.3	14.84	17.4
	35	36.1	25	15	13.9	5.1	12.0	19.5	16.8	22.85	30.1
	45($\varphi* = 39.9$)	44.6	31.7	20	15	7.2	17	29.2	35	31.37	43.8
7.0	25	34.6	23.1	12.3	10.2	35.3	13.4	24.3	18.6	26.42	29.7
	35	57.2	41.7	26.5	24.1	46.5	20	32.8	31.2	38.94	50.3
	45($\varphi* = 39.9$)	69.7	53.6	39	37	60	30	47.5	56	54.38	78.1

H = Helical

P = Plate

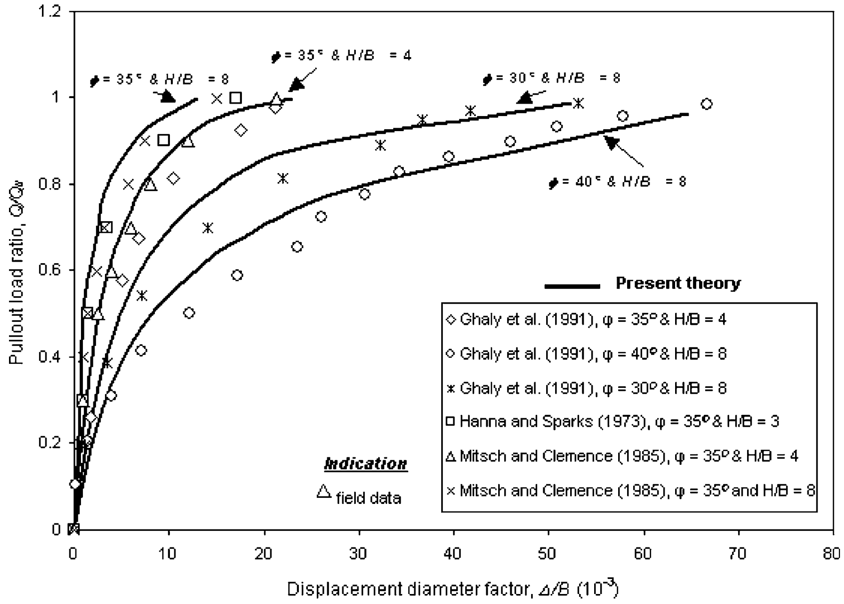


Fig. 10 Comparison between theoretical load–displacement curves and experimental results

or plate, determine the critical depth (H_c) using Eq. 42. The anchor is considered to be shallow if its depth (H) is less than H_c.

2. Use Fig. 3 to estimate the value of (a), knowing the values of φ and the ratio H/B.
3. Use Eq. 44 to calculate the radius of the influence circle in the sand mass at the ground surface, R. The anchors will perform as singles anchors if they are placed at a distance S, where: $S \geq 2R$. If however $S \leq 2R$, interaction between individual anchors may take place, designer may exercise the option to adjust spacing between anchors or to determine the group efficiency.
4. Determine the values of k_1 and k_2 from Table 1.
5. Knowing the unit weight of sand (γ), estimate the uplift capacity of the anchor (Q_u) using Eq. 33 or from the design charts given in Figs. 7a and b.
6. For the load–displacement relationship, calculate the value of K using Eq. 40.
7. Knowing K and Q_u, the load–displacement relationship of the anchor can be established using Eq. 36.

It is important to note that, for the case of anchors in dense sand it is recommended to use the modified angle of shear resistance, $\varphi*$, which can be determined from Eq. 49.

6 Conclusion

The cases of single vertical plate and helical anchors in sand have been investigated. The following conclusions were drawn:

1. The observed failure mechanism of Ghaly and Hanna (1991), was used to develop theories to predict the pullout capacity and the load–displacement characteristics for these anchors using the limit equilibrium method of analysis.
2. Empirical expression is presented to determine the critical depth of anchors, which separates the shallow from the deep. The critical depth depends on the diameter of the anchor base and the angle of shearing resistance of the sand.
3. Empirical expression is presented to determine the radius of influence on the ground surface of individual anchors. Accordingly, spacing between anchors can be established to avoid overlap of the stress zones of these anchors. In case of overlap, group action should be taken into consideration in determining the capacity of individual anchors.
4. The predicted values by the proposed theory compared well with the experimental and field data of single and multi helix, plate anchors and belled piles in loose and medium-dense sand. Discrepancies were noted between the calculated

and the measured results for these anchors in dense sand, which believed due to the dilatancy effect at shallow depths. In this case, it is recommended to use the modified angles of shearing resistance as described herein.

5. The predicted load–displacement curves for these anchors by the proposed theory compared well with the results produced by other theories available in the literature.
6. Design charts and design procedure are presented for practical purposes.

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